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# A very general quartic double fourfold or fivefold is not stably rational

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Abstract

Applying an idea of C. Voisin, we prove that a double cover of $\mathbb{P}^4$ or $\mathbb{P}^5$ branched along a very general quartic hypersurface is not stably rational.

## 1. Introduction

A projective variety $X$ is *stably rational* if $X \times \mathbb{P}^m$ is rational for some integer $m$. A stably rational variety is unirational; that the converse does not hold was shown by Artin and Mumford [AM72]. Their example is a double covering $X$ of $\mathbb{P}^3_{\mathbb{C}}$ branched along a quartic *symmetroid*, a surface defined by the vanishing of a symmetric 4-by-4 determinant of linear forms. They prove that the torsion subgroup of $H^3(X,\mathbb{Z})$ is nonzero, whereas it is trivial for stably rational varieties.

Unfortunately this method applies only to rather particular varieties, and not to natural families like Fano threefolds, complete intersections, etc. A more powerful approach was discovered recently by Voisin [Voi15]: the existence of torsion in $H^3(X,\mathbb{Z})$ implies the nontriviality of a certain Chow group, a property which behaves better under specialization. She obtained the following beautiful consequence.

Theorem (Voisin). *A double cover of $\mathbb{P}^3_{\mathbb{C}}$ branched along a very general quartic surface is not stably rational.*

Here "very general" means that the surface lies outside the union of countably many strict subvarieties in the space of quartic surfaces in $\mathbb{P}^3$.

The aim of this paper is to extend this result in higher dimension, as follows.

Theorem 1. *For $n = 4$ or $5$, a double cover of $\mathbb{P}^n_{\mathbb{C}}$ branched along a very general quartic hypersurface is not stably rational.*

These varieties are easily seen to be unirational (Proposition 5); to our knowledge they provide the first examples of prime (that is, with Picard number 1) Fano manifolds of dimension greater than 3 which are unirational but not rational.

To prove Theorem 1 we apply Voisin's method, as extended in [CTP14]. The following is the statement that we will use ([CTP14, Théorème 1.12] and [Voi15, Remark 1.3]).

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Proposition 1. *Let $B$ be a smooth complex variety, let $\mathrm{o}$ be a closed point of $B$, and let $f\colon \mathcal{X} \to B$ be a flat, projective morphism such that the generic fiber of $f$ is smooth and that the fiber $X := \mathcal{X}_{\mathrm{o}}$ is integral and admits a desingularization $\sigma\colon \tilde{X} \to X$ with the following properties:*

a) *The torsion subgroup of $H^3(\tilde{X},\mathbb{Z})$ is non trivial.*
b) *The fiber of $\sigma$ over any point $x \in X$ is a smooth rational variety over the residual field $\kappa(x)$.*

*Then for a very general point $b \in B$, the fiber $\mathcal{X}_b$ is not stably rational.*

We stress that condition b) must hold for all points of the scheme $X$, not only for closed points. Actually, Proposition 1 gives the (possibly) stronger result that $\mathcal{X}_b$ is not *retract rational*. Thus in Theorem 1 one can replace "stably rational" by "retract rational".

Voisin's theorem follows at once from Proposition 1 by taking for $X$ the Artin–Mumford example. To treat the higher-dimensional case, we simply take the obvious generalization of that example, namely a double covering $X \to \mathbb{P}^n$ branched along a quartic symmetroid. The variety $X$ is singular, but admits for $n = 4$ or 5 a simple desingularization[1] which satisfies condition b) of Proposition 1 (see Proposition 3). To check condition a), we view $\mathbb{P}^n$ as a linear system $L$ of quadrics in $\mathbb{P}^3$; then the smooth part $X_{\mathrm{sm}}$ of $X$ parametrizes the quadrics of $L$ of rank at least 3 together with the choice of a system of generatrices. The generatrices in each system are parametrized by $\mathbb{P}^1$, so we get a $\mathbb{P}^1$-bundle over $X_{\mathrm{sm}}$; this provides a 2-torsion class in $H^3(X_{\mathrm{sm}},\mathbb{Z})$. We will prove that this class comes from a nontrivial torsion class in $H^3(\tilde{X},\mathbb{Z})$ (Proposition 4), whence the result.

## 2. Linear systems of quadrics

**2.1.** Let $\mathcal{Q}$ be the linear system of quadrics in $\mathbb{P}^3_{\mathbb{C}}$. We denote by $\mathcal{Q}_i \subset \mathcal{Q}$ the subvariety of quadrics of rank at most $i$. We recall some basic properties of these varieties (see, for instance, [Vai82]):

- We have $\mathcal{Q} \cong \mathbb{P}^9$, the subvariety $\mathcal{Q}_3$ is a quartic hypersurface in $\mathcal{Q}$, $\dim \mathcal{Q}_2 = 6$, and $\dim \mathcal{Q}_1 = 3$.
- The singular locus of $\mathcal{Q}_i$ is $\mathcal{Q}_{i-1}$.
- The tangent cone $TC_q(\mathcal{Q}_3)$ at a point $q$ of $\mathcal{Q}_2 \setminus \mathcal{Q}_1$ is a rank 3 quadric in $T_q(\mathcal{Q})$.

**2.2.** For $n = 3, 4$ or 5, let $L$ be an $n$-dimensional projective subspace of $\mathcal{Q}$. We assume that $L$ does not meet $\mathcal{Q}_1$ and is transverse to $\mathcal{Q}_2$—this is the case if $L$ is sufficiently general. We put

$$\Delta := L \cap \mathcal{Q}_3 \quad \text{and} \quad \Sigma := L \cap \mathcal{Q}_2\,.$$

Thus $\Delta$ is a quartic hypersurface in $L$, with singular locus $\Sigma$ that is smooth, of dimension $n-3$. The tangent cone $TC_q(\Delta)$ at a point $q$ of $\Sigma$ is a rank 3 quadric in $T_q(L)$ (that is, a cone over a smooth conic, with vertex a linear space of dimension $n-3$).

**2.3.** Let $b\colon \tilde{L} \to L$ be the blow-up of $L$ along $\Sigma$; let $E$ be the exceptional divisor, and let $\tilde{\Delta}$ be the strict transform of $\Delta$.

[1] The desingularization becomes more complicated for $n \geqslant 6$; see Subsection 5.1.

Proposition 2. *The strict transform $\tilde{\Delta}$ is smooth and intersects $E$ transversally, so that $C := \tilde{\Delta}\cap E$ is smooth. Locally over $\Sigma$ for the Zariski topology, the embedding $C \hookrightarrow E$ is isomorphic to the embedding $C_0\times\Sigma \hookrightarrow \mathbb{P}^2\times\Sigma$, where $C_0$ is a smooth conic in $\mathbb{P}^2$.*

*Proof.* The fibration $E\to\Sigma$ is the projectivization of the normal bundle $N(\Sigma/L) = TL_{|\Sigma}/T\Sigma$, while the fibration $C\to\Sigma$ is the projectivization of the normal cone $NC(\Sigma/\Delta) = TC(\Delta)_{|\Sigma}/T\Sigma$ (observe that at each point $q$ of $\Sigma$, the tangent space $T_q\Sigma$ is the vertex of the tangent cone $TC_q(\Delta)$). By Subsection 2.2, the fibration $C\to\Sigma$ is a smooth conic bundle. Since $C$ is a Cartier divisor in $\tilde{\Delta}$, this implies that $\tilde{\Delta}$ is smooth along $C$, and therefore everywhere.

There is another natural $\mathbb{P}^1$-bundle over $\Sigma$: Let $C'\subset\mathbb{P}^3\times\Sigma$ be the variety of pairs $(x,q)$ with $x\in\operatorname{Sing}(q)$. The projection $C'\to\Sigma$ is a $\mathbb{P}^1$-bundle, with fiber $\operatorname{Sing}(q)$ above $q\in\Sigma$. It is easy to see that it is locally trivial for the Zariski topology. In fact, writing $\mathbb{P}^3=\mathbb{P}(V)$, we have a "universal quadric" $q_L\in H^0(L,\operatorname{Sym}^2V^*\otimes\mathcal{O}_L(1))$ over $L$, or equivalently a symmetric map $q_L^\sharp\colon V\otimes\mathcal{O}_L\to V^*\otimes\mathcal{O}_L(1)$. The kernel of $q^\sharp_{L|\Sigma}$ is a rank 2 vector bundle $K$ on $\Sigma$, and we have $C'=\mathbb{P}_\Sigma(K)$. We will now compare the $\mathbb{P}^1$-bundles $C$ and $C'$.

The projective tangent cone $\mathbb{P}TC_q(\Delta)$ to $\Delta$ at a singular point $q$ can be viewed as the variety of lines in $L$ passing through $q$ and intersecting $\Delta$ with multiplicity at least 3. Let $r\in L$; we denote by $\dot{q}$ and $\dot{r}$ quadratic forms defining $q$ and $r$, respectively. The line $\langle q,r\rangle$ belongs to $\mathbb{P}TC_q(\Delta)$ if and only if $\det(\dot{q}+t\dot{r})$ is divisible by $t^3$. Choose a decomposition $V=W\oplus\operatorname{Sing}(q)$. Then $\dot{q}+t\dot{r}$ is represented by a block matrix

$$\left(\begin{array}{c|c} \dot{q}_{|W}+t\dot{r}_{|W} & t(\dots) \\ \hline t(\dots) & t\dot{r}_{|\operatorname{Sing}(q)}\end{array}\right),$$

with $\det(\dot{q}_{|W})=\lambda\neq 0$. Thus $\det(\dot{q}+t\dot{r}) = t^2\,\lambda\det(\dot{r}_{|\operatorname{Sing}(q)}) \pmod{t^3}$, so the above condition is equivalent to $\det(\dot{r}_{|\operatorname{Sing}(q)})=0$, that is, to the quadric $r$ being tangent to $\operatorname{Sing}(q)$.

Similarly, the line $\langle q,r\rangle$ belongs to $\mathbb{P}T_q(\Sigma)$ if and only if all 3-by-3 minors of $\dot{q}+t\dot{r}$ are divisible by $t^2$; this is equivalent to $r$ containing the line $\operatorname{Sing}(q)$. Thus we have a canonical identification of the projectivization of the normal cone $TC_q(\Delta)/T_q(\Sigma)$ with $\operatorname{Sing}(q)$, mapping a line $\langle q,r\rangle$ not tangent to $\Sigma$ to the point of contact of $r$ with $\operatorname{Sing}(q)$. This shows that the $\mathbb{P}^1$-bundle $C$ is isomorphic to $C'$, hence locally trivial for the Zariski topology.

Finally, put $N := N(\Sigma/L)$; let $p$ be the projection $C\to\Sigma$. The embedding $i\colon C\hookrightarrow E = \mathbb{P}_\Sigma(N)$ is determined by the line bundle $M := i^*\mathcal{O}_E(1)$ and the surjective homomorphism $p^*N^*\to M$. The latter gives by adjunction an isomorphism $N^*\xrightarrow{\sim} p_*M$, so $i$ is isomorphic to the embedding $C\hookrightarrow\mathbb{P}_\Sigma\big((p_*M)^*\big)$.

Let $q\in\Sigma$. Replacing $\Sigma$ by a Zariski open subset containing $q$, we may assume that $p$ is the projection $\mathbb{P}^1\times\Sigma\to\Sigma$ and that $M$ is the pull-back of $\mathcal{O}_{\mathbb{P}^1}(2)$. Then $p_*M\cong\mathcal{O}_\Sigma^3$, and $i$ is isomorphic to the embedding $C_0\times\Sigma\hookrightarrow\mathbb{P}^2\times\Sigma$ in a Zariski neighborhood of $q$. □

## 3. The double covering

Let $\pi\colon X\to L$ be the double covering of $L$ branched along the quartic hypersurface $\Delta$. The variety $X$ is singular along $\pi^{-1}(\Sigma)$. The class $b^*(\Delta)-2E$ of $\tilde{\Delta}$ in $\operatorname{Pic}(\tilde{L})$ is divisible by 2, hence we can form the double covering $\tilde{X}\to\tilde{L}$ branched along $\tilde{\Delta}$. It gives a resolution $\sigma\colon\tilde{X}\to X$ of $X$, which is an isomorphism outside $\Sigma$; the variety $Q:=\sigma^{-1}(\Sigma)$ is a double covering of $E$ branched along $C$.

PROPOSITION 3. *The resolution $\sigma$ induces a smooth quadric fibration $Q \to \Sigma$, locally trivial for the Zariski topology. In particular, for any $q \in \Sigma$ the fiber $\sigma^{-1}(q)$ is a smooth quadric, rational over $\kappa(q)$.*

*Proof.* Let $q \in \Sigma$. In view of Proposition 2, replacing $\Sigma$ by a Zariski open subset containing $q$, we may assume that $Q$ is a double covering of $\mathbb{P}^2 \times \Sigma$ branched along $C_0 \times \Sigma$. Such a double covering is determined by the branch locus $C_0 \times \Sigma$ and a line bundle $M$ on $\mathbb{P}^2 \times \Sigma$ such that $M^{\otimes 2} \cong \mathrm{pr}_1^* \mathcal{O}_{\mathbb{P}^2}(2)$. Shrinking $\Sigma$ again, we may assume $M \cong \mathrm{pr}_1^* \mathcal{O}_{\mathbb{P}^2}(1)$; then the covering $Q \to \mathbb{P}^2 \times \Sigma$ is isomorphic to $Q_0 \times \Sigma \to \mathbb{P}^2 \times \Sigma$, where $Q_0 \to \mathbb{P}^2$ is the double covering of $\mathbb{P}^2$ branched along $C_0$. Since $Q_0$ is a smooth quadric, this implies the proposition. $\square$

This gives us condition b) of Proposition 1; we now check condition a).

PROPOSITION 4. *The 2-torsion subgroup of $H^3(\tilde{X}, \mathbb{Z})$ is nontrivial.*

*Proof.* We will prove that the Brauer group $\mathrm{Br}(\tilde{X})$ [Gro68] contains a nonzero 2-torsion element. This group injects into $H^2(\tilde{X}, \mathcal{O}_h^*)$, where $\mathcal{O}_h$ is the sheaf of holomorphic functions on $\tilde{X}$. Since $H^2(\tilde{X}, \mathcal{O}_{\tilde{X}}) = 0$, the latter group injects into $H^3(\tilde{X}, \mathbb{Z})$ by the exponential exact sequence, so this implies the proposition. $\square$

We put $U := \tilde{X} \setminus Q$. Let $\mathbb{G} := \mathbb{G}(2,4)$ denote the Grassmannian of lines in $\mathbb{P}^3$. We consider the incidence variety $I := \{(\ell, q) \in \mathbb{G} \times (L \setminus \Sigma) \mid \ell \subset q\}$ and the projection $p\colon I \to L \setminus \Sigma$.

The fiber $p^{-1}(q)$ is a disjoint union of two rational curves for $q \in L \setminus \Delta$ and is a single rational curve for $q \in \Delta \setminus \Sigma$. Therefore $p$ factors as a $\mathbb{P}^1$-fibration $\varphi\colon I \to U$, followed by the double covering $U \to L \setminus \Sigma$. The $\mathbb{P}^1$-bundle $\varphi$ gives a 2-torsion class $[\varphi]$ in the Brauer group $\mathrm{Br}(U)$ [Gro68]. We claim that this class is nonzero.

Suppose first $n = 3$. If the class of $\varphi$ in $\mathrm{Br}(U)$ were zero, the $\mathbb{P}^1$-bundle $\varphi\colon I \to U$ would be a projective bundle. But $I$ is a rational variety, because the projection $I \to \mathbb{G}$ is birational [Bea83, § 9], and we know that $\tilde{X}$ is not stably rational [AM72]. We conclude that $[\varphi] \neq 0$ in $\mathrm{Br}(U)$.

For $n = 4$ or 5, we choose a general 3-dimensional projective subspace $L'$ in $L$, and construct the corresponding subvarieties $U' \subset U$ and $I' := \varphi^{-1}(U')$. The class $[\varphi]$ in $\mathrm{Br}(U)$ restricts to $[\varphi_{|I'}]$ in $\mathrm{Br}(U')$, which is nonzero by the above; thus we find $[\varphi] \neq 0$ in $\mathrm{Br}(U)$ in all cases. We conclude with the following lemma.

LEMMA. *The restriction map $\mathrm{Br}(\tilde{X}) \to \mathrm{Br}(U)$ is an isomorphism.*

*Proof.* We consider the quadric fibration $f\colon Q \to \Sigma$. The two systems of generatrices of each fiber form a double covering of $\Sigma$ which is locally trivial for the Zariski topology (Proposition 3), hence trivial. We choose one of the two systems. In each fiber the generatrices of this system are parametrized by $\mathbb{P}^1$ and form a $\mathbb{P}^1$-fibration $g\colon G \to \Sigma$. For each point $x$ of $Q$ there is a unique generatrix of our system passing through $x$; this gives again a $\mathbb{P}^1$-fibration $h\colon Q \to G$ such that $g \circ h = f$. By Proposition 3 both fibrations are locally trivial for the Zariski topology, hence are projective bundles.

We claim that we can blow down $\tilde{X}$ along the fibers of $h$, more precisely, that there exist a compact complex manifold $\bar{X}$, a map $p\colon \tilde{X} \to \bar{X}$ and an embedding $G \hookrightarrow \bar{X}$ such that the diagram

$$\begin{array}{ccc} Q & \hookrightarrow & \tilde{X} \\ \downarrow{\scriptstyle h} & & \downarrow{\scriptstyle p} \\ G & \hookrightarrow & \bar{X} \end{array}$$

is obtained by blowing up $G$ in $\bar{X}$. According to the Fujiki–Nakano criterion [FN71], it suffices to prove that the restriction of the line bundle $\mathcal{O}_{\tilde{X}}(Q)$ to a fiber $\ell := h^{-1}(q)$ of $h$ has degree $-1$.

We have $K_{Q_{|\ell}} \cong K_{Q_q{}_{|\ell}} \cong \mathcal{O}_\ell(-2)$ . Recall that $\tilde{X}$ was obtained by first taking the blow-up $b\colon \tilde{L} \to L$ of $L$ along $\Sigma$, with exceptional divisor $E$, and then taking the double covering $d\colon \tilde{X} \to \tilde{L}$ branched along the surface $\tilde{\Delta} \in |b^*\Delta - 2E|$. Then

$$K_{\tilde{L}} \cong b^*\mathcal{O}_L(-n-1)(2E) \quad \text{and} \quad K_{\tilde{X}} \cong d^*b^*\mathcal{O}_L(-n+1)(Q)\,;$$

since $\ell$ is contracted by $b\circ d$, we find $K_{\tilde{X}|\ell} \cong \mathcal{O}_{\tilde{X}}(Q)_{|\ell}$. Using the adjunction formula we get $K_{Q_{|\ell}} \cong K_{\tilde{X}}(Q)_{|\ell} \cong \mathcal{O}_{\tilde{X}}(2Q)_{|\ell}$, therefore $\deg \mathcal{O}_{\tilde{X}}(Q)_{|\ell} = -1$, whence our claim.

Now, we have a commutative diagram

$$\begin{array}{ccc} H^2(\bar{X},\mathbb{Q}/\mathbb{Z}) & \longrightarrow & H^2(U,\mathbb{Q}/\mathbb{Z}) \\ \downarrow & & \downarrow \\ \mathrm{Br}(\bar{X}) & \longrightarrow & \mathrm{Br}(U)\,, \end{array}$$

where the vertical arrows are surjective; the top horizontal arrow is bijective by the Gysin exact sequence, because $G$ has codimension 2 in $\bar{X}$. Therefore the restriction map $\mathrm{Br}(\bar{X}) \to \mathrm{Br}(U)$ is surjective. Since it is the composition of $p^*\colon \mathrm{Br}(\bar{X}) \to \mathrm{Br}(\tilde{X})$ and the restriction map $\mathrm{Br}(\tilde{X}) \to \mathrm{Br}(U)$, it follows that the latter is surjective; but it is also injective since both Brauer groups inject into the Brauer group of the function field $\mathbb{C}(\tilde{X})$; see [Gro68, II, Corollaire 1.10]. □

Thus our desingularization $\tilde{X} \to X$ satisfies the conditions stated in Proposition 1. Theorem 1 follows by taking for $B$ the space of quartic hypersurfaces in $L = \mathbb{P}^n$, for $o \in B$ the point corresponding to $\Delta$, and for $\mathcal{X}$ the family of double coverings of $\mathbb{P}^n$ branched along those hypersurfaces.

## 4. Unirationality

The following result is classical for $n = 3$, and the proof extends easily to the general case.

PROPOSITION 5. *A double covering of $\mathbb{P}^n$ branched along an integral quartic hypersurface is unirational.*

*Proof.* Let $\pi\colon X \to \mathbb{P}^n$ be the double covering, and let $\mathbb{G}$ be the Grassmannian of lines in $\mathbb{P}^n$. Consider the variety $X^* \subset X \times \mathbb{G}$ of pairs $(x,\ell)$ with $\pi(x) \in \ell$. The projection $p\colon X^* \to X$ is a projective $\mathbb{P}^{n-1}$-bundle, with fiber at $x \in X$ the space of lines passing through $\pi(x)$.

For $(x,\ell)$ general in $X^*$, the curve $E := \pi^{-1}(\ell)$ is a smooth genus 1 curve in $X$ passing through $x$; there is a unique point $f(x,\ell) \in E$ such that the divisors $\pi^*q + f(x,\ell)$, for $q \in \ell$, are linearly equivalent to $3x$. This defines a rational map $f\colon X^* \dashrightarrow X$.

Let $(y,\ell)$ be a general point of $X^*$. On the genus 1 curve $\pi^{-1}(\ell)$ there are nine points $x$ such that $3x$ is linearly equivalent to $\pi^*q + y$ for $q \in \ell$, that is, such that $f(x,\ell) = y$. Thus $f$ is dominant, and a general fiber $f^{-1}(y)$ has dimension $n-1$; in particular, we have $p(f^{-1}(y)) \subsetneq X$.

Let $P$ be a general 2-plane in $\mathbb{P}^n$, and let $\tilde{P} := \pi^{-1}(P) \subset X$. We consider the restriction $f_P$ of $f$ to $p^{-1}(\tilde{P})$. We have $p(f_P^{-1}(y)) = \tilde{P} \cap p(f^{-1}(y)) \subsetneq \tilde{P}$. The projection $p\colon f_P^{-1}(y) \to \tilde{P}$ is generically injective (if $f(x,\ell) = y$, then $\ell = \langle \pi(x), \pi(y) \rangle$). Thus $\dim f_P^{-1}(y) \leqslant 1$. It follows that $f_P\colon p^{-1}(\tilde{P}) \dashrightarrow X$ is dominant. But $p^{-1}(\tilde{P})$ is a projective bundle over the rational surface $\tilde{P}$, hence is rational, and $X$ is unirational. □

## 5. Questions

**5.1.** It might be possible to extend our main result in dimension $n = 6, \ldots, 9$, by taking a general linear system $L \subset \mathcal{Q}$ of dimension $n$. However, for $n \geqslant 6$ this linear system contains rank 1 quadrics, which produce triple points of $\Delta$. The desingularization becomes much more intricate; we do not know whether the conditions a) and b) of Proposition 1 still hold.

**5.2.** In [CTP14] the authors show that a very general quartic threefold is not stably rational, by applying Proposition 1 to a singular quartic birational to the Artin–Mumford threefold. This has been extended by Totaro [Tot15] to very general hypersurfaces of degree at least $2\lceil (n+2)/3 \rceil$ in $\mathbb{P}^{n+1}$, in particular to quartic fourfolds, by combining Proposition 1 with an earlier method of Kollár. It would be interesting to extend the result to very general quartic fivefolds, which are known to be unirational [CM98].

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